QUIZ 19 SOLUTIONS: LESSON 26
NOVEMBER 2, 2018

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [10 pts] A rectangular box with a square base is to be constructed from materials that cost \$10 per square foot for the sides, top, and bottom. What is the minimum cost to construct a box with a volume of 1,000 cubic feet?

Recall that the method of LaGrange multipliers requires solving the following system of equations:

$$\begin{aligned} f_x &= \lambda g_x \\ f_y &= \lambda g_y \\ g(x,y) &= C \end{aligned}$$

where f is the function we are minimizing and $g(x,y) = C$ is a constraint.

<u>Solution</u>: We want to minimize the cost of the box. Since the box has a square base, our variables will be the width w and the height h. Then the cost function is given by

$$C(w,h) = 10(2w^2 + 4wh) = 20w^2 + 40wh.$$

We are told that the volume $V = w^2h$ is 1,000 cubic feet. This is the constraint. We write

$$\begin{aligned} 40w + 40h &= \lambda(2wh) \\ 40w &= \lambda(w^2) \\ w^2h &= 1,000 \end{aligned}$$

By the second equation, we see that $\lambda = \dfrac{40}{w}$. Plugging this into the first equation, we get

$$40w + 40h = \underbrace{\left(\frac{40}{w}\right)}_{\lambda}(2w) = 40(2h)$$

$$\Rightarrow \quad w + h = 2h$$

$$\Rightarrow \quad w = h$$

By the last equation,

$$w^2h = w^3 = 1,000 \Rightarrow w = 10.$$

We conclude that $h = w = 10$.

Therefore, the minimum cost of the box is

$$C(10,10) = 20(10)^2 + 40(10)(10) = 20(100) + 40(100) = 60(100) = \boxed{\$\ 6,000}.$$